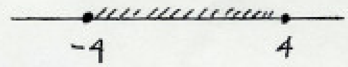
ONE GRAPHING PROBLEM

Pg 208

22 Graph $y = x\sqrt{16-x^2}$

(a) Domain: Domain is all x's such that $16-x^2 \geqslant 0$ i.e.



(b) Critical Points: $f'(x) = x \cdot \frac{1}{\cancel{2}}(16-x^2)^{-\frac{1}{2}}(-\cancel{2}x) + \sqrt{16-x^2} \cdot 1$

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$\therefore \quad f'(x) = \dfrac{-x^2}{\sqrt{16-x^2}} + \dfrac{\sqrt{16-x^2}}{1} = \dfrac{-x^2 + (16-x^2)}{\sqrt{16-x^2}} = \dfrac{16-2x^2}{\sqrt{16-x^2}} = \dfrac{2(8-x^2)}{\sqrt{16-x^2}}$

$\therefore \quad f'(x) = \dfrac{2(2\sqrt{2}-x)(2\sqrt{2}+x)}{\sqrt{16-x^2}}$

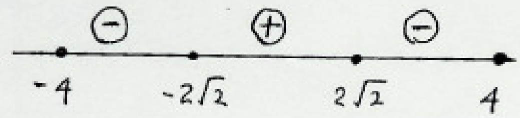
$\therefore$ $f'(x) = 0 \Leftrightarrow x = \pm 2\sqrt{2}$ (in the domain)

$f'(x)$ d.n.e $\Leftrightarrow x = \pm 4$ (in the domain)

$\therefore$ The critical points are $x = \pm 2\sqrt{2}$ and $x = \pm 4$

(c) Increasing/Decreasing intervals:

Sign of f':



$\therefore$ f is increasing on $[-2\sqrt{2}, 2\sqrt{2}]$

f is decreasing on $[-4, -2\sqrt{2}] \cup [2\sqrt{2}, 4]$

(d) Local Extrema: f has a local minimum at $x = -2\sqrt{2}$ pt: $(-2\sqrt{2}, -8)$

f has a local maximum at $x = 2\sqrt{2}$ pt: $(2\sqrt{2}, 8)$

(e) Concavity and points of inflection:

$$f'(x) = \frac{2(8-x^2)}{\sqrt{16-x^2}}$$

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$\therefore \quad f''(x) = 2 \cdot \dfrac{\sqrt{16-x^2}\cdot(-2x) - (8-x^2)\cdot\frac{1}{\cancel{2}}(16-x^2)^{-\frac{1}{2}}(-\cancel{2}x)}{(16-x^2)} = 2 \cdot \dfrac{\left[-2x\sqrt{16-x^2} + \dfrac{x(8-x^2)}{\sqrt{16-x^2}}\right]\cdot\sqrt{16-x^2}}{(16-x^2)\cdot\sqrt{16-x^2}}$

$f''(x) = \dfrac{2\left[-2x(16-x^2) + x(8-x^2)\right]}{(16-x^2)^{3/2}} = \dfrac{2x\left[-32+2x^2+8-x^2\right]}{(16-x^2)^{\frac{3}{2}}} = \dfrac{2x(x^2-24)}{(16-x^2)^{3/2}}$

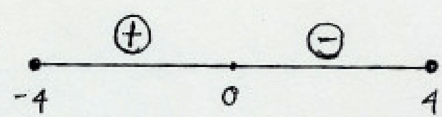
$\therefore \quad f''(x) = \dfrac{2x(x+\sqrt{24})(x-\sqrt{24})}{(16-x^2)^{3/2}}$

$\therefore$ $f''(x) = 0 \Leftrightarrow x = 0$ (in domain); $x = \pm\sqrt{24}$ (not in domain)

$f''(x)$ d.n.e $\Leftrightarrow x = \pm 4$ (in domain).

$\therefore$ The hypercritical points are $x = 0$, $x = \pm 4$.

Sign of f'':



$\therefore$ f is concave up on $[-4, 0]$

f is concave down on $[0, 4]$

f has a pt. of inflection at $x = 0$ pt: $(0, 0)$

(f) Intercepts: Y-int: Set $x = 0 \Rightarrow y = 0$ $\therefore$ $(0,0)$ is the Y-int.

X-int: Set $y = 0 \Rightarrow x\sqrt{16-x^2} = 0$ $\therefore$ $x = 0$ or $\underline{16-x^2 = 0}$
$x = \pm 4$.

$\therefore$ The X-int are $(0,0)$, $(-4,0)$ and $(4,0)$.

(f) Intercepts: Y-int: Set $x = 0 \Rightarrow y = 0$ $\therefore$ $(0, 0)$ is the Y-int.

x-int: Set $y = 0 \Rightarrow x\sqrt{16 - x^2} = 0$ $\therefore$ $x = 0$ or $\underbrace{16 - x^2 = 0}_{x = \pm 4.}$

$\therefore$ The X-int are $(0,0)$, $(-4,0)$ and $(4,0)$.

(g) Vertical and Horizontal Asymptotes:

There are no vertical asymptotes (because there is no x-value "a" which $\lim_{x \to a^{\pm}} f(x) = \pm\infty$)

Horiz. Asymps: $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ both do not exist.

Therefore, there are no horizontal asymptotes either.

GRAPH: