LOGARITHM FUNCTIONS, EXPONENTIAL FUNCTIONS, AND INVERSE TRIG FUNCTIONS

I LOGARITHM FUNCTIONS:

(a) $\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\longleftrightarrow$ (a) $\int \frac{1}{x}\,dx = \ln|x| + C$

(b) $\frac{d}{dx}\left(\ln f(x)\right) = \frac{f'(x)}{f(x)}$ $\longleftrightarrow$ $\boxed{\text{(b) } \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C}$ VER IMPOR

Note that $\ln x$ is the same as the $\underbrace{\text{logarithm}}_{\text{natural log}}$ $\log_e x$ where

$e \approx 2.71828$............(e is called the base of the natural logarithm)

Here are the five properties of logarithm from an algebra class:

1. $\log_a(M) + \log_a(N) = \log_a(MN)$
2. $\log_a(M) - \log_a(N) = \log_a\left(\frac{M}{N}\right)$
3. $\log_a(M^k) = k\log_a(M)$
4. $\log_a a = 1$
5. $\log_a 1 = 0$

II EXPONENTIAL FUNCTIONS:

The exponential function $y = e^x$ can be defined as the inverse (<u>NOT</u> the reciprocal) function of the natural logarithm function $y = \ln x$

(a) $\frac{d}{dx}(e^x) = e^x$ $\longleftrightarrow$ (a) $\int e^x\,dx = e^x + C$

(b) $\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$ $\longleftrightarrow$ $\boxed{\text{(b) } \int f'(x)e^{f(x)}\,dx = e^{f(x)} + C}$

VERY IMPORTANT

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III INVERSE TRIGONOMETRIC FUNCTIONS: $\underbrace{\text{arcSin}x}_{}$ $\underbrace{\text{arcCos}x}_{}$ $\underbrace{\text{arcTan}x}_{}$

The inverse trigonometric functions are $\text{Sin}^{-1}x$, $\text{Cos}^{-1}x$, $\text{Tan}^{-1}x$, etc...

(a) $\dfrac{d}{dx}\left(\text{Sin}^{-1}x\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\longleftrightarrow$ (a) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \text{Sin}^{-1}(x) + C$

(a)' $\dfrac{d}{dx}\left(\text{Sin}^{-1}\left(\dfrac{x}{a}\right)\right) = \dfrac{1}{\sqrt{a^2-x^2}}$ $\longleftrightarrow$ $\boxed{\text{(a)' } \displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \text{Sin}^{-1}\left(\frac{x}{a}\right) + C}$ ✱

(b) $\dfrac{d}{dx}\left(\text{Tan}^{-1}x\right) = \dfrac{1}{1+x^2}$ $\longleftrightarrow$ (b) $\displaystyle\int \frac{1}{1+x^2}\,dx = \text{Tan}^{-1}x + C$

(b)' $\dfrac{d}{dx}\left(\text{Tan}^{-1}\left(\dfrac{x}{a}\right)\right) = \dfrac{a}{a^2+x^2}$ $\longleftrightarrow$ $\boxed{\text{(b') } \displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\text{Tan}^{-1}\left(\frac{x}{a}\right) + C}$ ✱

<u>Note:</u> Derivatives of other inverse trig functions can be found on page 38