The Cross Product of 2 Vectors in Space

Defn: Suppose that $\underline{u} = u_1\underline{i} + u_2\underline{j} + u_3\underline{k}$ and $\underline{v} = v_1\underline{i} + v_2\underline{j} + v_3\underline{k}$ are two vectors in space. Then we define the cross product of $\underline{u}$ and $\underline{v}$ as the vector

$$\underline{u} \times \underline{v} = (u_2v_3 - u_3v_2)\underline{i} - (u_1v_3 - u_3v_1)\underline{j} + (u_1v_2 - u_2v_1)\underline{k}$$

COMMENTS: ① The cross product of 2 vectors is Always a vector.

② A convenient way of thinking of the cross pdt. (pretend it is "like a 3x3 determinant")

i.e: $$\underline{u} \times \underline{v} = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \end{array}$$

$$= \underline{i}(u_2v_3 - u_3v_2) - \underline{j}(u_1v_3 - u_3v_1) + \underline{k}(u_1v_2 - u_2v_1)$$

Example: Given $\underline{u} = -2\underline{i} + 3\underline{j} + \underline{k}$ and $\underline{v} = 4\underline{i} - 2\underline{j} - 3\underline{k}$, find $\underline{u} \times \underline{v}$.

$$\underline{u} \times \underline{v} = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -2 & 3 & 1 \\ 4 & -2 & -3 \end{vmatrix} \end{array} = \underline{i}(-9+2) - \underline{j}(6-4) + \underline{k}(4-12)$$

$$= -7\underline{i} - 2\underline{j} - 8\underline{k}$$

$$\therefore \underline{u} \times \underline{v} = -7\underline{i} - 2\underline{j} - 8\underline{k} = \langle -7, -2, -8 \rangle$$