Properties of the Cross Product (ALGEBRAIC)

Let $\underline{u}$, $\underline{v}$ and $\underline{w}$ be 3 vectors in space, and let c be any scalar. Then we have,

① $\underline{u} \times \underline{v} = -(\underline{v} \times \underline{u})$ [Cross Product is anti-commutative]

② $\underline{u} \times (\underline{v} + \underline{w}) = (\underline{u} \times \underline{v}) + (\underline{u} \times \underline{w})$ [Cross Product is distributive over vector addition]

③ $c(\underline{u} \times \underline{v}) = (c\underline{u}) \times \underline{v} = \underline{u} \times (c\underline{v})$

④ $\underline{u} \times \underline{0} = \underline{0} \times \underline{u} = \underline{0}$

* ⑤ $\underline{u} \times \underline{u} = \underline{0}$

⑥ $\underline{u} \cdot (\underline{v} \times \underline{w}) = (\underline{u} \times \underline{v}) \cdot \underline{w}$

More Properties of the Cross Product (Geometric)

Let $\underline{u}$ and $\underline{v}$ be any 2 nonzero vectors in space, and let θ be the angle between $\underline{u}$ and $\underline{v}$, $0 \leq \theta \leq \pi$.

* ① $\underline{u} \times \underline{v}$ is orthogonal to both $\underline{u}$ and $\underline{v}$

② $\|\underline{u} \times \underline{v}\| = \|\underline{u}\| \|\underline{v}\| \sin\theta$

③ $\underline{u} \times \underline{v} = 0 \iff \underline{u}$ and $\underline{v}$ are scalar multiples of each other

** ④ $\|\underline{u} \times \underline{v}\|$ = area of the parallelogram having $\underline{u}$ and $\underline{v}$ as adjacent sides