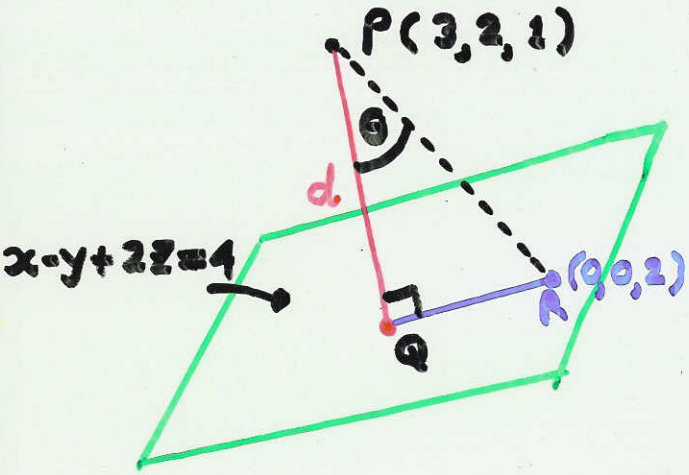
The (Minimum) Distance from a Point to a Plane

Example: Find the distance from $(3,2,1)$ to the plane $x-y+2z=4$



Let Q be the foot of the perpendicular from P to Q.

We need just to find PQ (=d)

First, find ANY point R on the plane. For example,

$x=0, y=0 \Rightarrow 0-0+2z=4 \Rightarrow z=2$

$\therefore$ R $(0,0,2)$ is a point on the plane.

Note that PQ is $\perp^r$ QR. From the right $\triangle$ PQR, $d = |\|\vec{PR}\|.\cos\theta| = \|\vec{PR}\|\,|\cos\theta|$ — ①

But $\vec{PQ}$ (or $\vec{QP}$) is along a normal vector to the plane, and we know $\underline{n} = \langle 1,-1,2\rangle$ is a normal vector.

$$\therefore |\cos\theta| = \frac{|\vec{PR}\cdot\underline{n}|}{\|\vec{PR}\|\,\|\underline{n}\|} = \frac{|\langle -3,-2,1\rangle.\langle 1,-1,2\rangle|}{\sqrt{9+4+1}.\sqrt{1+1+4}} = \frac{|-3+2+2|}{\sqrt{14}\sqrt{6}}$$

$$\therefore |\cos\theta| = \frac{1}{\sqrt{84}}$$

$\therefore$ By ①, $d = \sqrt{14}\cdot\frac{1}{\sqrt{84}} = \frac{1}{\sqrt{6}}$

$\therefore$ The required dist is $\frac{1}{\sqrt{6}}$ units.