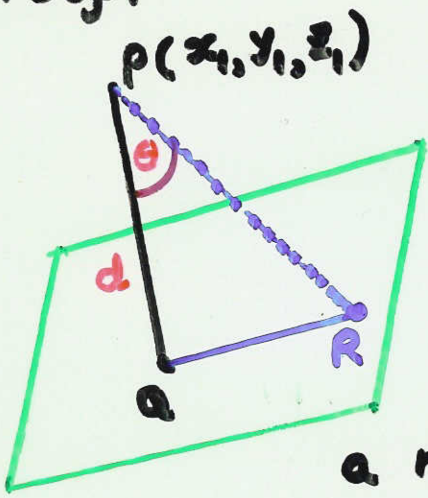
Now, we can do the above calculation and obtain the following more general result:

Result: The distance d from the point $P(x_1, y_1, z_1)$ to the plane $ax+by+cz+d=0$ is given by

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$$

proof:



Let $R(\alpha, \beta, \gamma)$ be any pt. on the plane

Note that this means

$$a\alpha + b\beta + c\gamma + d = 0 \quad \text{——— } (1)$$

Then $\overrightarrow{PR} = \langle \alpha - x_1, \beta - y_1, \gamma - z_1 \rangle$

$$d = \|\overrightarrow{PR}\| \, |\cos\theta| \quad \text{——— } (2)$$

a normal vector to the plane $= \underline{n} = \langle a, b, c \rangle$

Also, $$|\cos\theta| = \frac{|\overrightarrow{PR} \cdot \underline{n}|}{\|\overrightarrow{PR}\| \, \|\underline{n}\|} \quad \text{——— } (3)$$

By (2) & (3): $$d = \|\cancel{\overrightarrow{PR}}\| \frac{|\overrightarrow{PR} \cdot \underline{n}|}{\|\cancel{\overrightarrow{PR}}\| \, \|\underline{n}\|} = \frac{|\overrightarrow{PR} \cdot \underline{n}|}{\|\underline{n}\|}$$

$$\therefore \; d = \frac{|a(\alpha - x_1) + b(\beta - y_1) + c(\gamma - z_1)|}{\sqrt{a^2+b^2+c^2}}$$

By (1) above

$$= \frac{|(a\alpha + b\beta + c\gamma) - (ax_1 + by_1 + cz_1)|}{\sqrt{a^2+b^2+c^2}} = \frac{|-d - (ax_1 + by_1 + cz_1)|}{\sqrt{a^2+b^2+c^2}} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$$

Q.E.D