GRAD, DIV, CURL and all that !!

The operator ∇ (called 'del' or 'nabla') is defined as :

$$\nabla = \underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + \underline{k}\frac{\partial}{\partial z}$$

① Gradient of a function $f(x,y,z)$:

$$\text{grad } f(x,y,z) = \nabla f(x,y,z) = \underline{i}\frac{\partial f}{\partial x} + \underline{j}\frac{\partial f}{\partial y} + \underline{k}\frac{\partial f}{\partial z}$$

Note : f is a scalar function, but ∇f is a vector !

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② Divergence of a vector function $\underline{F}(x,y,z)$

where $\underline{F}(x,y,z) = M\underline{i} + N\underline{j} + P\underline{k}$

$$\text{div } \underline{F}(x,y,z) = \nabla \cdot \underline{F}(x,y,z) = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$$

Note: $\underline{F}$ is a vector function, but $\nabla \cdot \underline{F}$ is a SCALAR !!

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③ Curl of a vector function $\underline{F}(x,y,z)$

where $\underline{F}(x,y,z) = M\underline{i} + N\underline{j} + P\underline{k}$

$$\text{curl } \underline{F}(x,y,z) = \nabla \times \underline{F}(x,y,z) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \underline{i}\left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right) - \underline{j}\left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right) + \underline{k}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$$

Note: $\underline{F}$ is a vector function, and $\nabla \times \underline{F}$ is also a vector