Theorem (Properties of the Dot Product)

Suppose $\underline{u}$, $\underline{v}$ and $\underline{w}$ are any 3 vectors in plane or space. Let c be a scalar.

① Commutative Property of the Dot Product : $\underline{u}\cdot\underline{v} = \underline{v}\cdot\underline{u}$

② Distributivity of the Dot product over vector addition : $\underline{u}\cdot(\underline{v}+\underline{w}) = \underline{u}\cdot\underline{v} + \underline{u}\cdot\underline{w}$

③ $c(\underline{u}\cdot\underline{v}) = c\underline{u}\cdot\underline{v} = \underline{u}\cdot c\underline{v}$

④ $\underline{0}\cdot\underline{v} = 0$

⑤ Connection between the Dot Product and the magn. of a vector $\underline{v}\cdot\underline{v} = \|\underline{v}\|^2$

COMMENTS:

① Dot product of two vectors is ALWAYS a SCALAR

② Dot Product provides a very useful way of finding the angle between 2 vectors $\underline{u}$ and $\underline{v}$.

It is due to the result,

$$\cos\theta = \frac{\underline{u}\cdot\underline{v}}{\|\underline{u}\|\,\|\underline{v}\|}$$