THEOREM (Cauchy-Schwarz Inequality)

$\forall$ vectors $\underline{u}$ and $\underline{v}$ in 2D or 3D $|\underline{u}\cdot\underline{v}| \leq \|\underline{u}\|\|\underline{v}\|$

Proof: Let $\underline{u}$ and $\underline{v}$ be any 2 vectors on 2D or 3D

Then, $|\underline{u}\cdot\underline{v}| = \big|\, \|\underline{u}\|\|\underline{v}\| \cos\theta \,\big|$

$$= \big|\|\underline{u}\|\big|\,\big|\|\underline{v}\|\big|\,|\cos\theta|$$

$$= \|\underline{u}\|\,\|\underline{v}\|\,|\cos\theta|$$

$$\leq \|\underline{u}\|\,\|\underline{v}\| \cdot 1$$

($\because$ for any angle θ, $-1 \leq \cos\theta \leq 1$ or, $|\cos\theta| \leq 1$)

$\therefore |\underline{u}\cdot\underline{v}| \leq \|\underline{u}\|\|\underline{v}\|$

Q.E.D