

THE DEFINITION OF A GROUP

Defn: Suppose $\emptyset \neq G$, and suppose $*$ is a binary operation on G satisfying the following three conditions:

- (i) The operation $*$ is associative
- (ii) G has an identity element w.r.t $*$
- (iii) Every element of G has an inverse element

Then G is said to be a group w.r.t $*$. We will denote this group by $\langle G, * \rangle$

NOTE One can write the above group axioms (i)-(iii) in the following more precise manner:

- (i) $\forall x, y, z \in G \quad (x * y) * z = x * (y * z)$
- (ii) $\forall x \in G \quad \exists e \in G \quad \text{st} \quad x * e = x = e * x$
[e is called an id. elmt of G]
- (iii) $\forall x \in G \quad \exists y \in G \quad \text{st} \quad x * y = y * x = e$
[y is called an inverse element of x]