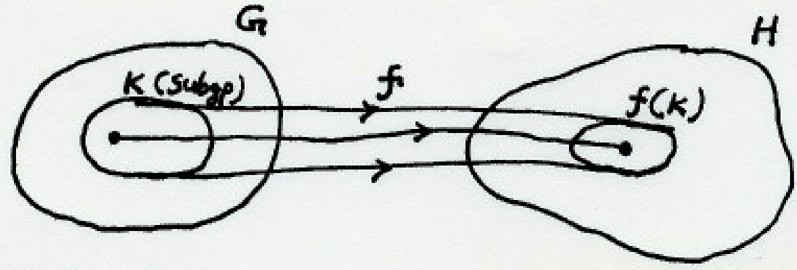
C3 (Pg 143) G and H are any 2 groups and $f: G \longrightarrow H$ is a homomorphism. K is any subgroup of G. Show that $f(K) = \{f(x) \mid x \in K\}$ is a subgroup of H.

<u>Proof:</u>



First $f(K) \neq \emptyset$, This is because Since K is a subgroup of G, we can say that $e_G \in K$. Therefore, $f(e_G) = e_H \in f(K)$.

It is clear that $f(K) \subseteq H$ (by defn of $f(K)$)

Now let $a, b \in f(K)$ be arbitrary.

[Need to show that $ab^{-1} \in f(K)$]

Since $a \in f(K)$, $a = f(u)$ for some $u \in K$.

Since $b \in f(K)$, $b = f(v)$ for some $v \in K$.

$\therefore$ $ab^{-1} = f(u) \cdot [f(v)]^{-1} = f(u) \cdot f(v^{-1})$ (f is a homo. & by thm 1 (ii), Pg 139)

$= f(uv^{-1})$ (By defn of a homomorphism)

$\therefore$ $ab^{-1} = f(uv^{-1})$

But since $u \in K$, $v \in K$, and K is a subgroup, we can say that $uv^{-1} \in K$ (by one thm for a subgroup)

$\therefore$ $ab^{-1} = f(uv^{-1})$ and $uv^{-1} \in K$.

$\therefore$ By defn of $f(K)$, $ab^{-1} \in f(K)$.

$\therefore$ $f(K)$ is a subgroup of H.

Q.E.D