D6 : Let us do the special case: (intersection of just TWO)
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Show that the intersection of any 2 normal subgroups is a normal subgroup.

Proof: Let G be any group, and let H, K be any two normal subgps of G. (i.e. $H \trianglelefteq G$ and $K \trianglelefteq G$)

Want to show that $H \cap K \trianglelefteq G$.

Recall by definition that $H \cap K = \{x \in G \mid x \in H \text{ and } x \in K\}$.

Earlier we have shown that $H \cap K$ is a subgroup of G. Review this part and include the same here to begin with.

Now to show the 'normality', let $a \in H \cap K$ and $x \in G$ be arbit.
[Need to show that $xax^{-1} \in H \cap K$].

First, since $a \in H \cap K$, we can say that $a \in H$. Since $a \in H$, $x \in G$, and $H \trianglelefteq G$, by defn of a normal subgroup, we have that $xax^{-1} \in H$ — ①

Secondly, since $a \in H \cap K$, we can say that $a \in K$. Since $a \in K$, $x \in G$, and $K \trianglelefteq G$, by defn of a normal subgroup, we have that $xax^{-1} \in K$ — ②

$\therefore$ By ① and ②, $xax^{-1} \in H \cap K$ (by definition of intersection of 2 sets)

$\therefore$ $\forall a \in H \cap K$ $\forall x \in G$, $xax^{-1} \in H \cap K$.

$\therefore$ By definition of a normal subgroup, $H \cap K$ is a normal subgp of G.

Q.E.D.

Note: A more general result is true: Let G be any gp and $\{N_\alpha\}_{\alpha \in L}$ be a family of normal subgroups of G. Then $\bigcap_{\alpha \in L} N_\alpha$ is a normal subgroup of G.