2 Related Problems on Subgroups.

① Same as prob C2, pg 49

Let G be any abelian group.

Let n be a <u>fixed</u> <u>integer.</u>

Let $H = \{x \in G \mid x^n = e\}$

Then show $H \leq G$

② This one is slightly different:

Let G be any abelian group.

Let $H = \{x \in G \mid x^n = e$ <u>for</u> <u>some</u> <u>$n \in \mathbb{Z}$</u>$\}$

Then show $H \leq G$

What I was trying to show was this one. <u>You can finish the details...</u>

Here is the complete solution for ①, the original problem. Remember we are working on an <u>Abelian group</u>, and this condition will be used.

o) First, $H \neq \emptyset$, Since $e \in H$. The fact that $e \in H$ is true because

$e^n = e$ for any $n \in \mathbb{Z}$.

Clearly $H \subseteq G$, by defn of H. $\therefore \emptyset \neq H \subseteq G$ (ie. H is a non empty subset of G).

i) Now let $x, y \in H$ be arbitrary. We want to show $xy^{-1} \in H$.

Since $x \in H$, by defn of H, $x \in G$ and $x^n = e$ ——— (α)

Since $y \in H$, by defn of H, $y \in G$, and $y^n = e$ ——— (β)

By (α) and (β), Since $x \in G$ and $y \in G$, we get that $xy \in G$. ($\because$ G is a group)

Now $(xy^{-1})^n = x^n(y^{-1})^n$ ($\because$ G is an <u>abelian gp</u> & by <u>Ex. H2, pg 43</u>)

$= x^n(y^n)^{-1}$

$= e.e^{-1}$ = ($\because$ by (α) and (β))

$= e$

$\therefore xy^{-1} \in G$ and $(xy^{-1})^n = e$. $\therefore$ By defn of H, $xy^{-1} \in H$. $\therefore H \leq G$.